A Unified Approach to Energy-Efficient Power Control in Large CDMA Systems

Farhad Meshkati, Dongning Guo, H. Vincent Poor, and Stuart C. Schwartz

Abstract

A unified approach to energy-efficient power control is proposed for code-division multiple access (CDMA) networks. The approach is applicable to a large family of multiuser receivers including the matched filter, the decorrelator, the linear minimum mean-square error (MMSE) receiver, and the (nonlinear) optimal detectors. It exploits the linear relationship that has been shown to exist between the transmit power and the output signal-to-interference-plus-noise ratio (SIR) in the large-system limit. It is shown that, for this family of receivers, when users seek to selfishly maximize their own energy efficiency, the Nash equilibrium is SIR-balanced. In addition, a unified power control (UPC) algorithm for reaching the Nash equilibrium is proposed. The algorithm adjusts the user's transmit powers by iteratively computing the large-system multiuser efficiency, which is independent of instantaneous spreading sequences. The convergence of the algorithm is proved for the matched filter, the decorrelator, and the MMSE receiver, and is demonstrated by means of simulation for an optimal detector. Moreover, the performance of the algorithm in finite-size systems is studied and compared with that of a conventional power control scheme, in which user powers depend on the instantaneous spreading sequences.

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Index Terms

Code-division multiple access (CDMA), energy efficiency, game theory, large systems, multiuser detection, multiuser efficiency, Nash equilibrium, power control.

I. Introduction

Power control is used for interference management and resource allocation in both the downlink and the uplink of code-division multiple access (CDMA) networks [1]–[8]. In particular, in the uplink each user transmits just enough power to achieve the required quality of service (QoS) without causing excessive interference in the network. In recent years, power control for multiuser receivers has attracted much attention due to their superior performance as compared to the single-user matched filter. For example, power control algorithms for the linear minimum mean-square error (MMSE) receiver and successive interference cancellation receivers have been proposed in [4] and [7], respectively. In the proposed schemes, the output signal-to-interference-plus-noise ratio (SIR) of each is measured and then the user's transmit power is adjusted to achieve the desired SIR.

Power control is conventionally modeled as an optimization problem under some quality of service (QoS) constraints. A practically appealing scheme is to minimize the total transmit power under the constraint that the output SIR of each user is above some lower bound. Under reasonable assumptions it is shown that the total transmit power is minimized when the SIR requirements are met with equality [1]. Another approach is to choose the transmit powers in such a way as to maximize the spectral efficiency (in bits/s/Hz) of the network, where the optimal strategy is essentially a water-filling scheme [6].

In recent years, game theory has been used to study power control in CDMA networks. In particular, power control can be modeled as a non-cooperative game in which each user selfishly maximizes its own utility. The strategy chosen by one user affects the performance of other users in the network through multiple-access interference (e.g., [9]–[15]).

Reference [14] studies the cross-layer design problem of joint power control and multiuser detection using game theory. The utility function in this case measures the number of bits

transmitted per joule of energy consumed, which is particularly suitable for energy-constrained networks. Focusing on linear receivers, the Nash equilibrium for the non-cooperative game has been derived. A Nash equilibrium refers to a set of strategies for which no user can unilaterally improve its own utility [16]. It is also shown in [14] that for all linear receivers, the transmit powers of the users are SIR-balanced at Nash equilibrium (i.e., all users have the same output SIR).

This work extends the results in [14] to a much larger family of receivers (including nonlinear ones) in the so-called *large-system* regime, where the number of users and the spreading factor are large with a given ratio. This is due to results in [17] where a linear relationship between the input power and the output SIR is shown to exist for a family of multiuser detectors in the large-system limit. Members of this family include well-known receivers such as the matched filter (MF), the decorrelator (DEC), the linear MMSE receiver, as well as the individually optimal (IO) and jointly optimal (JO) multiuser detectors.[1] By exploiting the linear relationship, which is characterized by the multiuser efficiency, we propose a unified power control (UPC) algorithm for reaching the Nash equilibrium. The convergence of the proposed algorithm is proved for linear receivers and is demonstrated by means of simulation for an optimal detector.

Since the UPC algorithm is based on large-system results, it does not depend on the instantaneous spreading sequences. The true SIR, however, does depend on the spreading sequences. Therefore, the SIR achieved by the UPC algorithm deviates from the target SIR as the spreading sequence changes from one symbol to the next. This in turn will result in a loss in the utility. The performance of the UPC algorithm in finite-size systems is studied and it is shown that if the spreading factor is reasonably large, the SIR achieved by the UPC algorithm stays close to the target SIR most of the time which means that the loss is insignificant.

The rest of the paper is organized as follows. Section II provides the system model and relevant results in multiuser detection. Section III discusses the game-theoretic approach to power control.

[1]The individually optimal detector minimizes the error probability of detecting an individual symbol whereas the jointly optimal detector minimizes the probability of error for detecting the entire vector of all users' symbols at a symbol interval [18]. The jointly optimal detector is often referred to as the maximum likelihood (ML) detector.

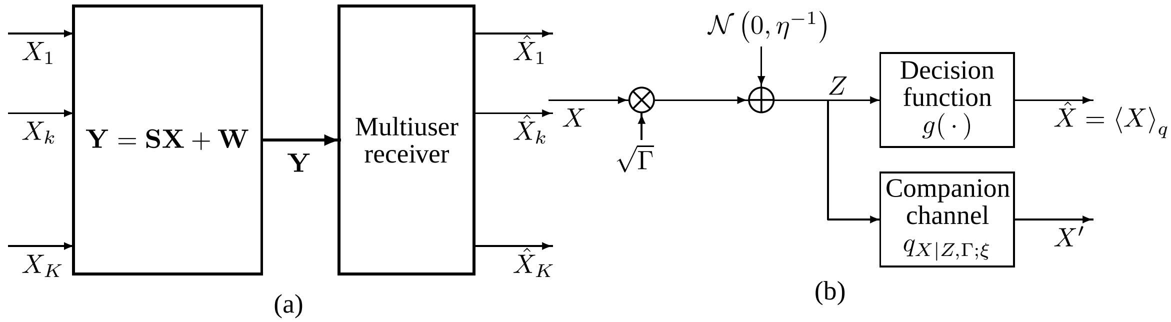

Fig. 1. (a) The DS-CDMA system with multiuser detection, where only the output for user k is shown. (b) The equivalent single-user Gaussian channel for user k; also shown is the companion channel.

The UPC algorithm for reaching Nash equilibrium is proposed and studied in Section IV. The performance of the UPC algorithm in finite-size systems is studied in Section V. Simulation results are presented in Section VI before conclusions are drawn in Section VII.

II. Multiuser Detection and Power Control

A. System Model and Multiuser Detection

Consider the uplink of a synchronous DS-CDMA system with K users and spreading factor N. Let p_k, h_k, and σ^2 represent the transmit power, channel gain and the background noise variance (including other-cell interference) respectively, for user k. The received signal-to-noise ratio (SNR) for user k is then

$$\Gamma_k = \frac{p_k h_k}{\sigma^2}. \tag{1}$$

The received signal (after chip-matched filtering) sampled at the chip rate over one symbol duration can be represented as

$$\mathbf{Y} = \mathbf{S}\mathbf{X} + \mathbf{W} = \sum_{k=1}^{K} \sqrt{\Gamma_k} X_k \mathbf{s}_k + \mathbf{W}, \tag{2}$$

where $\mathbf{s}_k$ and X_k are the spreading sequence and transmitted symbol of user k, respectively, and $\mathbf{W} \sim \mathcal{N}(0, \mathbf{I})$ is the noise vector consisting of independent standard Gaussian entries (see Fig. 1(a)). Random spreading sequence is assumed for all users, and the input symbols X_k are assumed to be independent and identically distributed (i.i.d.) with unit variance.

Suppose the receiver is a linear filter $\mathbf{c}_k$, the detection output can be represented as a sum of three independent components:

$$\hat{X}_k = \mathbf{c}_k^T \mathbf{Y} \tag{3}$$

$$= \sqrt{\Gamma_k} X_k + \mathsf{MAI}_k + V_k \tag{4}$$

i.e., the desired signal, the MAI and Gaussian background noise. The quality of the output is effectively measured in terms of the output SIR,

$$\gamma_k = \frac{p_k h_k (\mathbf{c}_k^T \mathbf{s}_k)^2}{\sigma^2 \mathbf{c}_k^T \mathbf{c}_k + \sum_{j \neq k} p_j h_j (\mathbf{c}_k^T \mathbf{s}_j)^2} = \eta_k \Gamma_k \tag{5}$$

because the multiple-access interference (MAI_k) is asymptotically Gaussian (e.g., [19]). The degradation in SNR due to the MAI is known as the *multiuser efficiency*, denoted by η_k. In fact, (4) can be regarded as an equivalent single-user Gaussian channel for user k as depicted in Fig. 1(b), in which the function $g(\cdot)$ is an identity mapping, e.g., $g(z) = z$, $\forall z$.

B. Power Control

We model power control as a non-cooperative game in which each user tries to maximize its own utility (see e.g., [9]–[15]). We follow [9] to define the utility of user k as

$$u_k = \frac{T_k}{p_k} \text{ bits/joule}, \tag{6}$$

where T_k is the throughput of user k, i.e., the net number of information bits delivered correctly per unit time (sometimes referred to as *goodput*). This utility function captures the tradeoff between throughput and battery life and is particularly suitable for applications where saving power is more important than achieving a high throughput.

The throughput for user k can be quantified as

$$T_k = \frac{L}{M} R f(\gamma_k), \tag{7}$$

where L and M are the number of information bits and the total number of bits in a packet, respectively (i.e., $L - M$ bits of overhead); R is the transmission rate and $f(\gamma_k)$ is the efficiency function representing the packet success rate. The underlying assumption is that if a packet is

in error, it will be retransmitted. The efficiency function, $f(\gamma_k)$, is assumed to be increasing and S-shaped[2] (sigmoidal) with $f(\infty) = 1$. We also require that $f(0) = 0$ to ensure that $u_k = 0$ when $p_k = 0$. These assumptions are valid in many practical systems (see [14] for further details). Combining (6) and (7), user k's utility is given by

$$u_k = \frac{L\,R\,f(\gamma_k)}{M\,p_k}\ . \tag{8}$$

Using a sigmoidal efficiency function, it can be shown that the utility function in (8) is quasi-concave.[3]

Suppose each user is allowed to vary its transmit power only in order to selfishly maximize the utility, the power control game is described as

$$\max_{p_k} u_k \quad \text{for} \quad k = 1, \cdots, K. \tag{9}$$

For this non-cooperative game, a Nash equilibrium is a set of transmit powers $(p_1, \ldots, p_K)$ for which no user can unilaterally improve its own utility (i.e., a stable state). It has been shown in [14] that the following proposition holds for all linear receivers.

Proposition 1: The utility-maximizing strategy for user k is given by $p_k^* = \min(\hat{p}_k, P_{max})$, where $\hat{p}_k$ is the transmit power that results in an output SIR equal to γ^*, which is the solution to $f(\gamma) = \gamma f'(\gamma)$, and P_{max} is the maximum allowed power. Furthermore, the proposed power control game has a unique Nash equilibrium.

This proposition implies that, at Nash equilibrium, the users' transmit powers are SIR-balanced. The key point in proving Proposition 1 is that for all linear receivers there is a linear relationship between the output SIR and transmit power of a user. If we take the derivative of the utility function in (6) with respect to the transmit power and equate it to zero, we have

$$p_k \frac{\partial \gamma_k}{\partial p_k} f'(\gamma_k) - f(\gamma_k) = 0. \tag{10}$$

[2]An increasing function is S-shaped if there is a point above which the function is strictly concave, and below which the function is strictly convex.

[3]A function is quasiconcave if there exists a point below which the function is non-decreasing, and above which the function is non-increasing.

Based on the expression of the output SIR (5), we can write

$$\frac{\partial \gamma_k}{\partial p_k} = \frac{\gamma_k}{p_k} \ . \tag{11}$$

Therefore, the utility of user k is maximized when $\gamma_k = \gamma^*$, the (positive) solution to

$$f(\gamma) = \gamma f'(\gamma) \ . \tag{12}$$

The existence of a Nash equilibrium is guaranteed because of the quasiconcavity of the utility function and its uniqueness is because of the one-to-one relationship between the user's transmit power and output SIR.

III. Large Multiuser Systems and Power Control

This paper studies power control in *large* systems. Mathematically, we consider the so-called *large-system limit*, where both the number of users and the spreading factor tend to infinity but with their ratio converging to a positive number, i.e., $K/N \rightarrow \alpha$. Since, in a large system, the dependence between the SNRs Γ_k is reasonably weak, we assumed that Γ_k are i.i.d. with distribution P_Γ at a given time. Moreover, P_Γ varies slowly over time as the timescale of power control is much larger than the symbol interval.

In general, the multiuser efficiency depends on the received SNRs, the spreading sequences as well as the type of detector. However, in the asymptotic case of large systems, the dependence on the spreading sequences vanishes and the received SNRs affect η only through their distribution. In particular, the multiuser efficiency of the matched filter and the decorrelator are obtained as

$$\eta^{mf} = \frac{1}{1+\alpha \mathbb{E}\{\Gamma\}} \tag{13}$$

$$\eta^{dec} = 1 - \alpha \quad \text{for} \quad \alpha < 1 \tag{14}$$

while the efficiency of the (linear) MMSE receiver is the unique solution to the following fixed-point equation

$$\frac{1}{\eta^{mmse}} = 1 + \alpha \mathbb{E}\left\{\frac{\Gamma}{1+\eta^{mmse}\Gamma}\right\} \tag{15}$$

where the expectation is over P_Γ.

The analysis of the SIR using (5) does not directly apply to nonlinear receivers because the output of such receivers cannot be decomposed as a sum of the desired signal and independent interference; neither is the output asymptotically Gaussian. Remarkably, reference [17] finds that, under mild assumptions, the output of a nonlinear receiver converges in the large-system limit to a simple monotone function of a "hidden" Gaussian statistic conditioned on the input, i.e.,

$$\hat{X}_k \to g\big(\sqrt{\Gamma_k}\, X_k + U_k\big) \tag{16}$$

where $U_k \sim \mathcal{N}\big(0, \eta^{-1}\big)$ is independent of X_k. By applying an inverse of this function to the detection output $\hat{X}_k$, an equivalent conditionally Gaussian statistic $Z_k = \sqrt{\Gamma_k}\, X_k + U_k$ is recovered. Each symbol X_k traverses an equivalent single-user Gaussian channel, so that the output SIR (defined for the equivalent Gaussian statistic Z_k) completely characterizes the system performance. This result is referred to as the "decoupling principle." The equivalent channel is illustrated in Fig. 1(b). Indeed, as far as the posterior probability $P_{X_k|\hat{X}_k}$ is concerned, the multiuser model (Fig. 1(a)) and the single-user model (Fig. 1(b)) are asymptotically indistinguishable.

A. *Posterior Mean Estimators*

The decoupling principle is shown in [17] to hold for a broad family of multiuser receivers, called the *posterior mean estimators* (PME). Given the observation $\mathbf{Y}$ and the spreading matrix $\mathbf{S}$, a PME computes the mean value of some posterior probability distribution $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$, which is conveniently denoted as

$$\langle \mathbf{X} \rangle_q = \mathbb{E}_q \left\{ \mathbf{X} \mid \mathbf{Y}, \mathbf{S} \right\} \tag{17}$$

where $\mathbb{E}_q\{\cdot\}$ stands for the expectation with respect to the measure q.

In this work, the posterior $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$ supplied to the PME is induced from the following postulated CDMA system,

$$\mathbf{Y} = \mathbf{S}\mathbf{X}' + \varrho \mathbf{W} \tag{18}$$

which differs from the actual channel (2) by only the input and the noise variance. In particular, the components of $\mathbf{X}'$ are i.i.d. with distribution q_X, and the postulated noise level ϱ serves as

a control parameter. The posterior $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$ is determined by q_X and $q_{\mathbf{Y}|\mathbf{X},\mathbf{S}}$ according to Bayes' formula

$$q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}(\mathbf{x}|\mathbf{y},\mathbf{S}) = \frac{q_{\mathbf{X}}(\mathbf{x})q_{\mathbf{Y}|\mathbf{X},\mathbf{S}}(\mathbf{y}|\mathbf{x},\mathbf{S})}{\int q_{\mathbf{X}}(\mathbf{x})q_{\mathbf{Y}|\mathbf{X},\mathbf{S}}(\mathbf{y}|\mathbf{x},\mathbf{S})\ \mathrm{d}\mathbf{x}}. \tag{19}$$

Indeed, the PME so defined is parameterized by (q_X, ϱ) and can be regarded as the optimal detector for the postulated multiuser system (18). In case the postulated posterior $q_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$ is identical to $p_{\mathbf{X}|\mathbf{Y},\mathbf{S}}$, the PME is a soft version of the individually optimal detector. The postulated posterior, however, can also be chosen such that the PME becomes one of many other detectors, including but not limited to the matched filter, decorrelator, linear MMSE receiver, as well as the optimal detectors. Thus the concept of PME is generic and versatile.

B. Decoupling Principle for PME

Let $q_{Z|X,\Gamma;\xi}$ represent the input–output relationship of a scalar Gaussian channel,

$$q_{Z|X,\Gamma;\xi}(z|x,\Gamma;\xi) = \sqrt{\frac{\xi}{2\pi}}\exp\left[-\frac{\xi}{2}\left(z-\sqrt{\Gamma}\,x\right)^2\right]. \tag{20}$$

Similar to that in the multiuser setting, by postulating the input distribution to be q_X, a posterior probability distribution $q_{X|Z,\Gamma;\xi}$ is induced from q_X and $q_{Z|X,\Gamma;\xi}$ using Bayes' formula (cf. (19)). Thus we have a single-user companion channel defined by $q_{X|Z,\Gamma;\xi}$, which outputs a random variable X' given the channel output Z (Fig. 1(b)). A (generalized) single-user PME is defined naturally as:

$$\langle X\rangle_q = \mathbb{E}_q\left\{X \mid Z,\Gamma;\xi\right\} \tag{21}$$

The probability law of the composite system depicted by Fig. 1(b) is determined by Γ, η and ξ.

Proposition 2 ([17]): Fix $(\alpha, P_\Gamma, p_X, q_X, \varrho)$. The joint probability distribution of $(X_k, \langle X_k\rangle_q)$ converges to the joint probability distribution of $\left(X, g\left(\sqrt{\Gamma_k}\,X+U\right)\right)$ where $U\sim\mathcal{N}\left(0,\eta^{-1}\right)$ and

$$g(z) = \mathbb{E}_q\left\{X \mid Z=z,\Gamma;\xi\right\}, \tag{22}$$

where the multiuser efficiency η satisfies together with ξ the coupled equations:

$$\eta^{-1} = 1 + \alpha\,\mathbb{E}\left\{\Gamma\cdot\mathcal{E}(\Gamma;\eta,\xi)\right\}, \tag{23a}$$

$$\xi^{-1} = \varrho^2 + \alpha\,\mathbb{E}\left\{\Gamma\cdot\mathcal{V}(\Gamma;\eta,\xi)\right\}, \tag{23b}$$

where the expectations are taken over P_Γ. Here we define the mean squared error of the PME as

$$\mathcal{E}(\Gamma;\eta,\xi) = \mathbb{E}\left\{ \left(X - \langle X\rangle_q \right)^2 \middle| \Gamma;\eta,\xi \right\}, \tag{24}$$

and also define the variance of the companion channel as

$$\mathcal{V}(\Gamma;\eta,\xi) = \mathbb{E}\left\{ \left(X' - \langle X\rangle_q \right)^2 \middle| \Gamma;\eta,\xi \right\}. \tag{25}$$

In case of multiple solutions to (23), (η,ξ) is chosen to minimize the free energy (see [17]).

Proposition 2 reveals that, from an individual user's viewpoint, the input–output relationship of the multiuser channel concatenated with the multiuser receiver is asymptotically identical to that of the scalar Gaussian channel with a (nonlinear) decision function. The key performance measure is the equivalent SIR η of the scalar channel, which is easy to compute by solving the fixed-point equation (23), since the functions $\mathcal{E}$ and $\mathcal{V}$ can be easily computed given the distribution of the transmitted symbols and the type of receiver; and the expectations are taken with respect to the received SNR distribution.

By choosing appropriate parameters (q_X, ρ), the PME can be made to represent the matched filter, the decorrelator, and the linear MMSE detector as well as the individually and jointly optimal detectors. The multiuser efficiencies of the linear receivers are given as (13)–(15). The multiuser efficiency of the individually optimal detector is found to satisfy the fixed-point equation

$$\frac{1}{\eta^{io}} = 1 + \alpha\mathbb{E}\left\{ \Gamma - \Gamma\int_{-\infty}^{+\infty} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} \tanh\left(\eta^{io}\Gamma - z\sqrt{\eta^{io}\Gamma}\right) \mathrm{d}z \right\}. \tag{26}$$

This result generalizes to multirate systems as well [20].

In principle, the multiuser efficiency may be different for different users with different types of receivers. As the uplink is the focus of this work, it is assumed that all users use the same type of receiver and hance are subject to the same multiuser efficiency.

C. Power Control

The decoupling principle implies that in large systems there is a linear relationship between the transmit power and the output SIR of each user, i.e.,

$$\gamma_k = \eta \Gamma_k \ , \tag{27}$$

where the multiuser efficiency η depends on the SNR distribution P_Γ rather than the individual Γ_k's. This is mainly due to the fact that in a large system, as one user's transmit power varies, the interference seen by the user essentially stays the same as long as the overall distribution of the received powers remains the same.

Even though (27) is a large-system result, it is a very good approximation for most finite-size systems of practical interest. The linear relationship (27) implies that, for the family of PME receivers, (11) is satisfied in large systems. Therefore, the argument given above for the linear receivers extends to a larger family of receivers that includes some nonlinear receivers such as the individually and joint optimal multiuser detectors.

This means that, in the asymptotic case of large systems, the Nash equilibrium for the power control game in (8) is SIR-balanced for all the receivers belonging to the family of PME for which (27) is satisfied. In other words, the Nash equilibrium is reached when each user transmits at a power level that achieves an output SIR equal to γ^*, the solution of $f(\gamma) = \gamma f'(\gamma)$. It is interesting to note that the Nash equilibrium SIR, γ^*, is independent of the type of receiver and depends only on physical-layer parameters such as modulation, coding and packet size.

Clearly, any deviation of a user's output SIR from γ^* results in a loss in the user's utility, or equivalently, its energy efficiency. For example, fixing other users' transmit powers, if the output SIR of user k is equal to $\hat{\gamma}$ (instead of γ^*), the resulting utility is given by

$$\hat{u}_k = \left(\frac{\gamma^* f(\hat{\gamma})}{\hat{\gamma} f(\gamma^*)} \right) u_k^* \ , \tag{28}$$

where u_k^* is the maximum utility for user k (corresponding to $\gamma_k = \gamma^*$).

IV. Unified Power Control Algorithm

In this section, we propose a unified power control algorithm for reaching the Nash equilibrium of the power control game, which is applicable to the family of PME receivers. The algorithm iteratively adjusts the transmit powers in order to reach an output SIR equal to γ^*. While we assume balanced SIRs here as a property of the Nash equilibrium, the UPC algorithm is also applicable to the case of unequal target SIRs.

The UPC Algorithm carries out the following iteration:

1) Let $n = 0$, start with initial powers $p_1(0), \cdots, p_K(0)$.
2) Use (23) and the power profile to compute the multiuser efficiency, $\eta(n)$.
3) For user k, update the powers according to

$$p_k(n+1) = \frac{1}{\eta(n)} \frac{\gamma^* \sigma^2}{h_k}. \tag{29}$$

4) *n*=*n*+1, stop if convergence; otherwise, go to Step 2.

In Step 2, while finding an analytical expression for the multiuser efficiency is difficult for most multiuser detectors, it can be easily obtained from (23) using numerical methods. Note that (23) need to be solved only once per iteration for each user. The uplink receiver (e.g., base station) can, for example, compute η and feed it back to the user terminal. The above algorithm is applicable to a large family of receivers which includes many popular receivers such as the matched filter, the decorrelator, and the MMSE detector as well as the individually and jointly optimal multiuser detectors. Note, for example, that for a matched filter receiver, the UPC algorithm becomes the same as the bilinear power control algorithm proposed in [21] for minimizing the SIR error. In fact, one may also extend Proposition 2 to the case that different users use different type of receivers and Algorithm 1 will apply by replacing η by individual efficiencies η_k.

It is apparently a dilemma that the success of the power control scheme depends on the assumption that the SNR distribution is fixed, while the purpose of power control is to adjust the powers which may affect the SNR distribution. This, however, can be resolved naturally in practice, where the number of users is finite, by an iterative power control algorithm and

by replacing the expectations in (23) with an average over all users' received SNRs (or their estimates). For example, (15) can be expressed as

$$\frac{1}{\eta} = 1 + \frac{\alpha}{K} \sum_{k=1}^{K} \frac{\Gamma_k}{1 + \eta \Gamma_k} \tag{30}$$

At any rate, the UPC scheme provides a viable mechanism to approach the Nash equilibrium.

There are certainly other practical issues that need further investigation. For example, the UPC algorithm requires the background noise and other-cell interference power to be measured. This could be done by silencing the users in a cell in a coordinated manner. In addition, while we will show in Section VI that the UPC algorithm converges very quickly, the convergence rate and effects of estimation/measurement errors on the convergence of the algorithm require further analysis.

We now prove the convergence of the UPC algorithm for the matched filter, the decorrelator, and the MMSE receiver.[4] Let $\mathbf{\Gamma} = [\Gamma_1, \cdots, \Gamma_K]$ and define an interference function,

$$I(\mathbf{\Gamma}) = \frac{\gamma^*}{\eta(\mathbf{\Gamma})} \ . \tag{31}$$

The dependence of the multiuser efficiency on $\mathbf{\Gamma}$ is explicitly shown here. By (29) and (31), the UPC algorithm can be expressed as

$$\Gamma_k(n+1) = I\left(\mathbf{\Gamma}(n)\right), \quad k = 1, \ldots, K. \tag{32}$$

Proposition 3: For the matched filter, the decorrelator, and the MMSE receiver, if there exists a $\hat{\mathbf{\Gamma}}$ such that $\hat{\Gamma}_k \geq I(\hat{\mathbf{\Gamma}})$, $k = 1, \ldots, K$, then for every initial vector $\mathbf{\Gamma}(0)$, the recursion (32) converges to the unique fixed point solution of $\Gamma_k^* = I(\mathbf{\Gamma}^*)$, $k = 1, \ldots, K$. Furthermore, for any feasible $\hat{\mathbf{\Gamma}}$ (i.e., $\hat{\Gamma}_k \geq I(\hat{\mathbf{\Gamma}})$ for all k), $\Gamma_k^* \leq \hat{\Gamma}_k$ for all k.

Proof: The existence of a $\hat{\mathbf{\Gamma}}$ implies that a feasible SNR vector exists for achieving γ^*. It suffices then to show that $I(\mathbf{\Gamma})$ is a standard interference function [3], i.e., for all $\mathbf{\Gamma}$ with $\Gamma_k \geq 0$ for all k, the following three properties are satisfied: 1) Positivity: $I(\mathbf{\Gamma}) > 0$; 2) Monotonicity: If

[4]The convergence analysis for a general receiver remains an open problem because (23) is difficult to work with.

$\Gamma'_k \geq \Gamma_k$ for all k, then $I(\mathbf{\Gamma}') \geq I(\mathbf{\Gamma})$; 3) Scalability: For all $\theta > 1$, $\theta I(\mathbf{\Gamma}) > I(\theta\mathbf{\Gamma})$. Evidently, it is equivalent to showing the three properties for $\hat{I}(\mathbf{\Gamma}) = 1/\eta(\mathbf{\Gamma})$.

Positivity of $\hat{I}(\mathbf{\Gamma})$ is trivial by (23a) for all receivers since $\eta \in [0, 1]$.

Consider first the matched filter. The multiuser efficiency is given by (13), where $\mathbb{E}\{\Gamma\} = \sum_{k=1}^{K} \Gamma_k/K$. If $\mathbf{\Gamma}' \geq \mathbf{\Gamma}$, then $\mathbb{E}\{\Gamma'\} \geq \mathbb{E}\{\Gamma\}$ and hence $\hat{I}(\mathbf{\Gamma}') \geq \hat{I}(\mathbf{\Gamma})$. To prove the third property, note that, for $\theta > 1$, $\hat{I}(\theta\mathbf{\Gamma}) = 1 + \alpha\mathbb{E}\{\theta\Gamma\} < \theta + \alpha\theta\mathbb{E}\{\Gamma\} = \theta\hat{I}(\mathbf{\Gamma})$.

Consider now the decorrelator, the multiuser efficiency of which is constant $\eta = 1 - \alpha > 0$ for $\alpha < 1$. Properties 2 and 3 are trivial.

For the MMSE receiver, the multiuser efficiency is the solution to (15), or equivalently, the unique solution of

$$\eta + \alpha\mathbb{E}\left\{\frac{1}{\frac{1}{\eta\Gamma} + 1}\right\} = 1. \tag{33}$$

Note that the left-hand side of (33) increases if both η and Γ increase. Thus if $\mathbf{\Gamma}' \geq \mathbf{\Gamma}$, we must have $\eta(\mathbf{\Gamma}') \leq \eta(\mathbf{\Gamma})$ to maintain the equality. Hence, $\hat{I}(\mathbf{\Gamma}') \geq \hat{I}(\mathbf{\Gamma})$. To prove the third property, let us define $\eta' = \eta(\theta\mathbf{\Gamma})$ and $\eta'' = \theta\eta'$, where $\theta > 1$. Therefore,

$$\eta'' + \alpha\theta\mathbb{E}\left\{\frac{1}{\frac{1}{\eta''\Gamma} + 1}\right\} = \theta. \tag{34}$$

Showing $\theta\hat{I}(\mathbf{\Gamma}) > \hat{I}(\theta\mathbf{\Gamma})$ is equivalent to showing $\eta'' > \eta$. Since the left-hand side of (33) is increasing in η, and

$$\eta'' + \alpha\mathbb{E}\left\{\frac{1}{\frac{1}{\eta''\Gamma} + 1}\right\} = 1 + \left(1 - \frac{1}{\theta}\right)\eta'' > 1, \tag{35}$$

we must have $\eta'' > \eta$. Therefore, $\theta\hat{I}(\mathbf{\Gamma}) > \hat{I}(\theta\mathbf{\Gamma})$. This completes the proof. ∎

V. Performance Evaluation and Discussion

The UPC algorithm described in Section IV is a large-system approach and hence independent of the spreading sequences. Therefore, after convergence, the transmit powers need not be updated as long as the channel gains remain static. Evidently, the actual SIRs depend on the spreading sequences (see for example (5)). Consequently, as the spreading sequence changes, the output SIRs achieved by the UPC algorithm fluctuate around the target SIR. This fluctuation results in a loss in energy efficiency (see (28)).

The question of interest is: if the UPC algorithm is used, how close will the SIRs be to the target SIR? In the following, we study the performance of the UPC algorithm for finite-size systems and compare it with that of an SIR-based algorithm. We focus on the decorrelator and the linear MMSE receiver.

A. The Decorrelator

For the decorrelator, it is sensible to assume $\alpha < 1$. The large-system multiuser efficiency is given by $\eta^{dec} = 1 - \alpha$. Hence the SNR dictated by the UPC algorithm is

$$\Gamma_k^* = \Gamma_{dec}^* = \frac{\gamma^*}{1-\alpha}, \quad k = 1, \cdots, K, \tag{36}$$

which should lead to an SIR of γ^* in the large-system limit.

However, the actual output SIR for a finite-size system is given by

$$\gamma_k = \left(\frac{\gamma^*}{1-\alpha} \right) \Big/ \left[\left(\tilde{\mathbf{S}}^T \tilde{\mathbf{S}} \right)^{-1} \right]_{kk}, \tag{37}$$

where $\tilde{\mathbf{S}} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K]$ and $[\cdot]_{kk}$ extracts the kth diagonal entry of a matrix.

It has been shown that in large systems, the distribution of $1/\left[(\mathbf{S}^T\mathbf{S})^{-1} \right]_{kk}$ can be approximated by a beta distribution with parameters $(N - K + 1, K - 1)$ [22]. As a result, the probability density function of γ_k is given approximately by

$$f_{\gamma_{dec}}(z) = \left(\frac{1}{\Gamma_{dec}^*} \right)^{N-1} \frac{z^{N-K} (\Gamma_{dec}^* - z)^{K-2}}{B(N-K+1, K-1)} \tag{38}$$

where $z \leq \Gamma_{dec}^*$ and $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} \mathrm{d}t$. Therefore, as the spreading sequences change from symbol to symbol, the probability that γ_k stays within Δ dB of γ^* is given by

$$P_{\Delta,dec} = \Pr \left\{ |\gamma_{dec}(\text{dB}) - \gamma^*(\text{dB})| \leq \Delta \right\} \tag{39}$$

$$= \int_{\gamma_L}^{\gamma_H} f_{\gamma_{dec}}(z) \mathrm{d}z \ , \tag{40}$$

where $\gamma_L = 10^{-\frac{\Delta}{10}} \gamma^*$ and $\gamma_H = 10^{\frac{\Delta}{10}} \gamma^*$.

Alternatively, the fluctuation of the actual SIR around γ^* can be approximated less accurately by a Gaussian distribution with variance [23]

$$\zeta^2 = \frac{2{\gamma^*}^2 \alpha}{(1-\alpha)N} \ , \tag{41}$$

i.e., $\gamma_{dec} \sim \mathcal{N}\left(\gamma^*, \zeta^2\right)$. Therefore, the probability that γ_k stays within Δ dB of γ^* is approximately given by

$$P^{\text{norm}}_{\Delta,dec} = \Phi\left(\frac{\gamma_H - \gamma^*}{\zeta}\right) - \Phi\left(\frac{\gamma_L - \gamma^*}{\zeta}\right), \tag{42}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard Gaussian distribution.

B. The MMSE Receiver

If the linear MMSE receiver is used and all users have the same target SIR, γ^*, the steady-state SNRs will be identical to $\Gamma^* = \gamma^*/\eta$ after the UPC algorithm converges, where the multiuser efficiency is given by

$$\eta = \frac{1-\alpha}{2} - \frac{1}{2\Gamma} + \frac{1}{2}\sqrt{(1-\alpha)^2 + \frac{2(1+\alpha)}{\Gamma} + \frac{1}{\Gamma^2}}. \tag{43}$$

It can be shown that the fluctuation of the true SIR around γ^* is approximately Gaussian with variance [23], [24]:

$$\zeta^2 = \frac{1}{N}\,\frac{2\gamma^{*2}}{1-\alpha\left(\frac{\gamma^*}{1+\gamma^*}\right)^2}\ . \tag{44}$$

The probability that γ_k stays within Δ dB of γ^* admits a similar expression to (42) using the function $\Phi(\cdot)$.

It is seen from these approximations that the variance of fluctuations of SIR decreases as $1/N$. In the following section, we demonstrate the performance of the UPC algorithm using simulations and also investigate the accuracy of the theoretical approximations discussed above.

VI. Numerical Results

Consider the uplink of a randomly spread DS-CDMA system with K users and spreading factor N. The noise variance is assumed to be $\sigma^2 = 1.6 \times 10^{-14}$. We use $f(\gamma) = (1 - e^{-\gamma})^M$ as the efficiency function[5] with $\gamma^* = 6.4$ (=8.1 dB).

We first demonstrate the convergence of the UPC algorithm assuming $K = 8$ and $N = 32$. The channel gain for user k is given by $h_k = 0.1 \times d_k^{-4}$ where d_k is the distance of user k from

[5]This is a useful example for the efficiency function and serves as an approximation to the packet success rate that is very reasonable for moderate to large packet sizes.

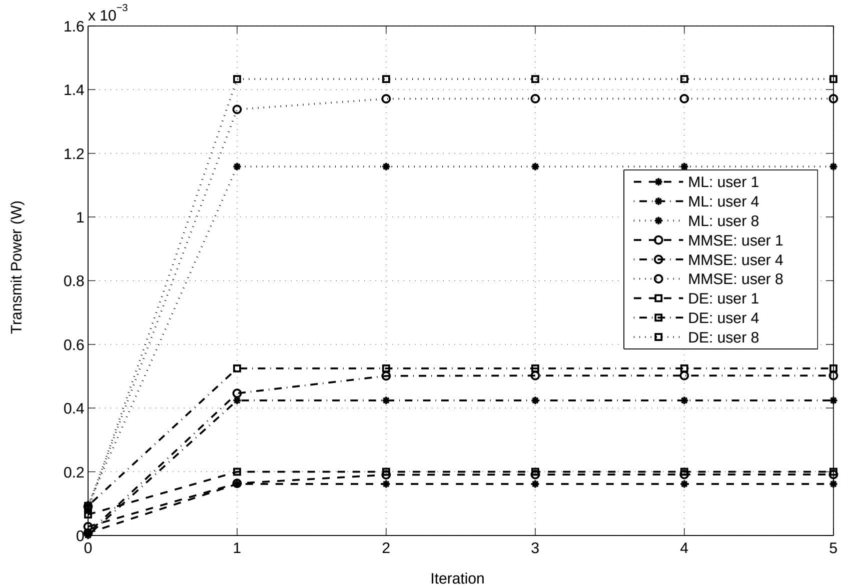

Fig. 2. Transmit powers for the ML, MMSE, and decorrelator, using the UPC algorithm ($N = 32$ and $K = 8$).

the uplink receiver (e.g., base station). Assume $d_k = 100 + 10k$ in meters. We implement the UPC algorithm for the decorrelator and the MMSE receiver as well as the maximum likelihood detector.

Fig. 2 plots the transmit powers for users 1, 4, and 8 at the end of each iteration. It is seen that for all three receiver types, the UPC algorithm converges quickly to steady-state values. The results are similar when the initial power values and/or K and N are changed. It is also observed that the steady-state transmit powers for the decorrelator and the MMSE receiver are close to those of the ML detector (in this case, the difference is less than 22%). This means that in terms of energy efficiency, which is quantified by the utility achieved at Nash equilibrium, the decorrelator and the MMSE receiver are almost as good as the ML detector.

We next investigate the fluctuation of the SIR and bit-error-rate (BER) achieved by the (large-system) UPC algorithm against perfect power control where the SIR is computed using instantaneous spreading sequences (labeled SIR-based in plots). Fig. 3 shows the SIR and bit-error-rate (BER) of user 1 using the MMSE detector. It is seen that the SIR-based algorithm achieves the target SIR, γ^*, at all time whereas the output SIR for the UPC algorithm fluctuates

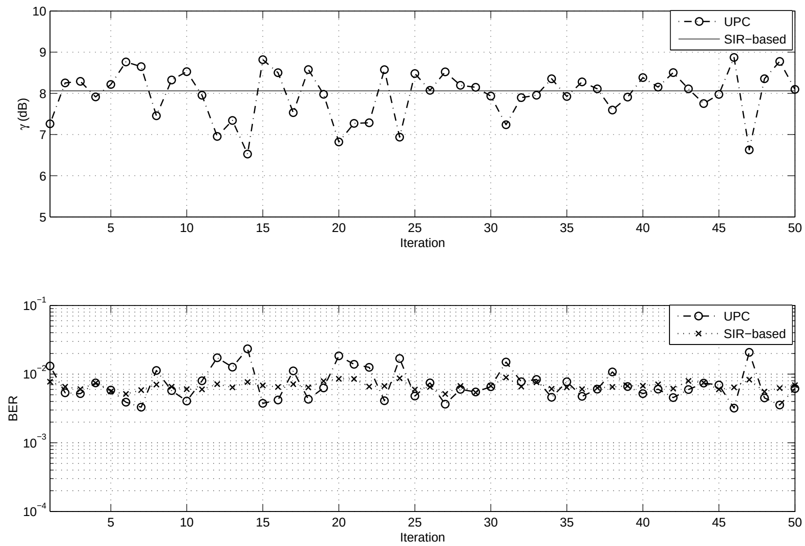

Fig. 3. User 1 output SIR and BER for the UPC algorithm and SIR-based algorithm with the MMSE receiver ($N = 32$ and $K = 8$).

around the target SIR as the spreading sequences change. Also, the fluctuations in the BER are larger when the UPC algorithm is used.

To evaluate the accuracy of the theoretical approximations given in Section V, Figs. 4 and 5 plot the cumulative distribution function (CDF) of the output SIR γ for the decorrelator and the MMSE receiver with different spreading factors and both low and high system loads. The plots show CDFs obtained from simulation (based on 100,000 realizations) as well as those predicted by the theoretical approximations given in Section V. It is seen from the figures that the theoretical approximations become more accurate as the spreading factor increases. Also, the approximations are generally more accurate when the system load is low. Note that, for the decorrelator, the approximation based on a beta distribution is slightly more accurate than the one based on a Gaussian distribution, especially for small spreading factors and large system loads.

To quantify the discrepancies between the simulation results and the theoretical approximations, we compute $P_{\Delta,dec}$ and $P_{\Delta,MMSE}$ using the CDFs obtained from simulations as well as those predicted by theory (see (39) and (42)). Table I shows the results for different spreading

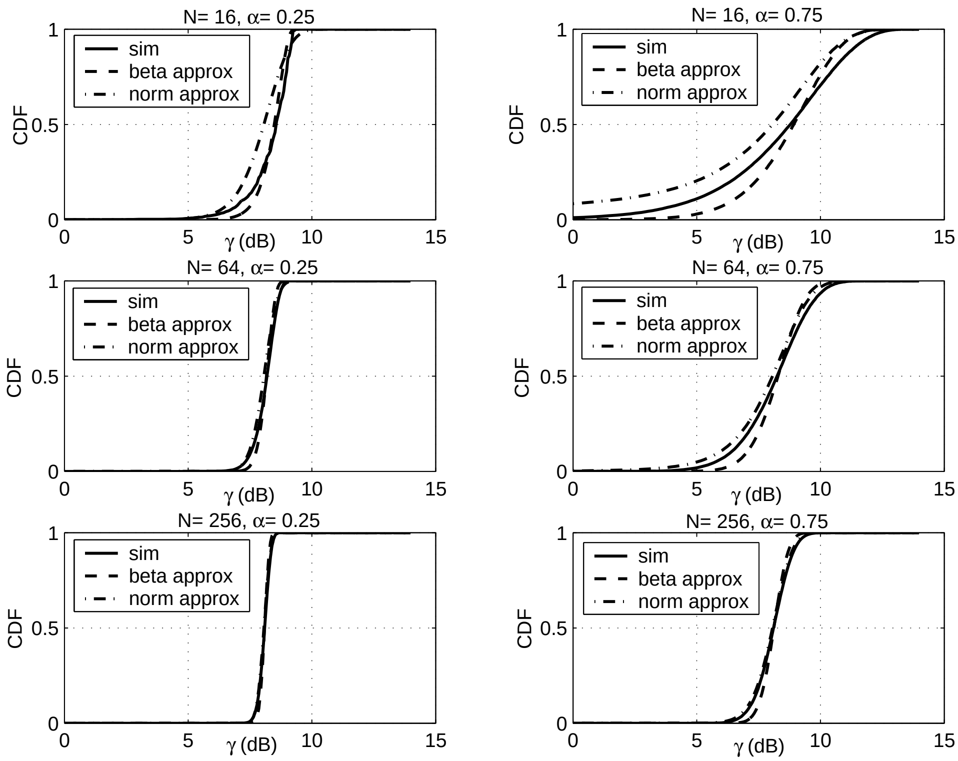

Fig. 4. CDFs of γ for the decorrelator.

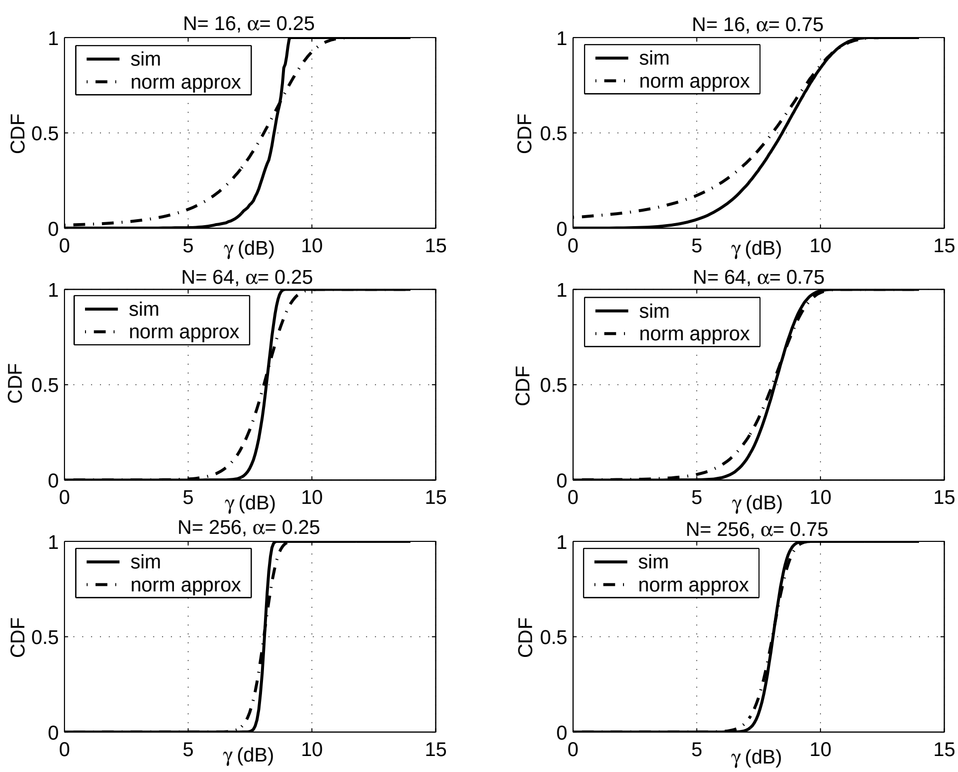

Fig. 5. CDFs of γ for the MMSE receiver.

TABLE I

SUMMARY OF RESULTS FOR THE DECORRELATOR AND THE MMSE RECEIVER

N	$P^{sim}_{1\text{dB},dec}$ $\alpha = 0.25$	$P^{beta}_{1\text{dB},dec}$ $\alpha = 0.25$	$P^{norm}_{1\text{dB},dec}$ $\alpha = 0.25$	$P^{sim}_{1\text{dB},dec}$ $\alpha = 0.75$	$P^{beta}_{1\text{dB},dec}$ $\alpha = 0.75$	$P^{norm}_{1\text{dB},dec}$ $\alpha = 0.75$	$P^{sim}_{1\text{dB},MMSE}$ $\alpha = 0.25$	$P^{norm}_{1\text{dB},MMSE}$ $\alpha = 0.25$	$P^{sim}_{1\text{dB},MMSE}$ $\alpha = 0.75$	$P^{norm}_{1\text{dB},MMSE}$ $\alpha = 0.75$
16	0.77	0.87	0.74	0.28	0.19	0.30	0.93	0.46	0.41	0.33
64	0.98	1.0	0.97	0.54	0.64	0.55	0.99	0.76	0.74	0.61
256	1.0	1.0	1.0	0.87	0.96	0.87	1.0	0.98	0.98	0.91

factors and system loads for $\Delta = 1$ dB. The numbers in the table represent the probability that γ is within 1 dB of γ^*. Based on (28), a 1-dB increase in the output SIR results in 10% loss in the user's utility. The probabilities obtained by simulation suggest that the UPC algorithm performs better for the MMSE receiver than for the decorrelator. It is also seen from the table that when the spreading factor is small, the fluctuations in the output SIR are considerable, especially when the system load in high. The performance improves as the spreading factor increases. For example, for the MMSE receiver, when $N = 256$ and $\alpha = 0.75$, the SIR stays within 1 dB of the target SIR 98% of the time. It is also observed that the theoretical approximations for the MMSE detector are pessimistic. For the decorrelator, while approximating the SIR by a beta distribution is more accurate (see Fig. 4), the values obtained for $P_{\Delta,MMSE}$ by the Gaussian approximation are closer to the simulation results. This is because the slope of the CDF of γ is closer to the slope of the Gaussian CDF. Since $P_{\Delta,MMSE}$ heavily depends on the slope of the CDF, it is more accurately predicted by the Gaussian approximation (rather than the beta approximation).

VII. CONCLUSION

A unified approach to energy-efficient power control in large systems is proposed, which is applicable to a large family of linear and nonlinear multiuser receivers. The approach exploits the linear relationship between the transmit power and the output SIR in large systems. Taking a non-cooperative game-theoretic approach with emphasis on energy efficiency, it is shown that the Nash equilibrium is SIR-balanced not only for linear receivers but also for some

nonlinear receivers such as the individually and jointly optimal multiuser detectors. In addition, a unified power control algorithm for reaching the Nash equilibrium is proposed. It would be straightforward to extend the unified approach to multirate and multicarrier systems based on related large-system results [20], [25].

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